

Ratio and Rates



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Common Core State Standards

The intent of this instructional sequence is to teach ratio and rates for understanding and in a manner consistent with the Common Core State Standards. Teaching in a standards-based environment means not only attending to the content but also the eight mathematical practices outlines in the Standards:

1. *Make sense of problems and persevere in solving them*
2. *Reason abstractly and quantitatively*
3. *Construct viable arguments and critique the reasoning of others*
4. *Model with mathematics*
5. *Use appropriate tools strategically*
6. *Attend to precision*
7. *Look for and make use of structure*
8. *Look for and express regularity in repeated reasoning*

Classroom environment. To accomplish these practices with this instructional sequence it is important to establish a classroom environment in which students feel obligated to explain their reasoning, ask questions when they do not understand, critique and understand the reasoning of others, and use mistakes as sites for learning opportunities. It is also imperative that the teacher poses problems and encourages students to persevere in solving them. The role of the teacher in this environment is to listen to students as they work on problems, encourage them to persevere when stuck, and orchestrate productive whole class discussions in which the mathematical ideas of the sequence become public for debate. The teacher does not tell students how to solve the problems, but rather encourages them to work either independently or with each other until they have some type of solution, whether right or wrong.

Modeling mathematics. The instructional sequence was designed so that students are introduced to negative numbers within the realistic context of finance. Throughout the sequence, the tasks and their wording change to encourage students to create their own models for reasoning about integers. The problems are sequenced to help students move from the concrete, realistic context of working with net worth, debts and assets toward the more quantitative, abstract, symbolic operations. The tasks encourage students to observe patterns and structures to curtail their thinking into more efficient operations. A vertical number line is presented as a tool for students to use throughout the sequence as a means of organizing and modeling their integer work.

Classroom Organization. Typically the class period is organized in a Launch, Explore, Discussion (LED) cycle or several cycles. The teacher begins the class period by launching the task for the day, typically taking less than five minutes.

The **launch** may consist of a bellwork, followed by the teacher reminding students of their discussion yesterday and ending with introducing the problem(s) for the day.

There are minutes set aside for **student exploration**, typically 5-20 minutes depending on the problem. We have found it helpful to allow for a few minutes of independent think-time followed by partner conversation, but any arrangement is fine. During explore time, the teacher's role is NOT to help students who are stuck find an answer. In fact, if students do not have a way the teacher should encourage them to work with a partner. The teacher's main role during explore time IS to collect data on the way students are solving the problem so that she can know how to lead the follow up discussion.

The class debriefs their work in a whole class **discussion** led by the teacher. Because the teacher has collected data on student reasoning, she knows who she will call on to present and in what order. Often times, the discussion session begins with the teacher listing all the different answers students have constructed to a problem on the board. She can then ask students to decide which answer or answers they believe are correct and then have the authors of those answers defend them. During this conversation the teacher is making sure that students are attempting to understand their classmate's arguments, ask questions when they do not understand, critique the solution, determine whether they have a different way, and contribute when they have a more sophisticated or efficient method.

There are other formats that can be just as effective as the LED cycle and teachers must decide which teaching formats have been effective in their own practice.

Big Ideas for the Instructional Sequence

<i>Big idea</i>	<i>Tools/imagery</i>	<i>Possible Topics of Discourse</i>	<i>Activity Pages</i>
Linking composite units	Connecting Pictures of aliens to food bars	If the rule is 1 food bar feeds 3 aliens, the rule can't be broken if we add more food bars	Page 1
Iterating linked composites	Informal symbolizing (e.g., tables, two columns of numbers, pictures of aliens and food bars)	How students keep track of two quantities while making them bigger	Pages 2-4
	Ratio table	Keeping track of two linked quantities while they grow additively	Pages 3-4
Additive versus multiplicative reasoning	Fold back to pictures; Shortened ratio table	Adding or multiplying to build up	Page 5
Structuring ratios multiplicatively	Shortened ratio tables through multiplication and division with scale factors	Efficient ways of curtailing long ratio tables What does the horizontal scale factor represent? What does the vertical scale factor represent?	Pages 3-7
Creating equivalent ratios	Ratio tables with missing values; Traditional proportion representation (two ratios separated with equal sign)	Adding versus multiplying; Meanings of scale factors; What do decimal scale factors mean?	Pages 8-10

Changing Contexts			
<i>Big idea</i>	<i>Tools/imagery</i>	<i>Possible Topics of Discourse</i>	<i>Activity Pages</i>
	Ratio tables	Adding versus multiplying; horizontal and vertical scale factors	Pages 11-12 and Pages 15-17
Analyzing equivalent ratios	“fraction” imagery	Reducing ratios; Vertical and horizontal scale factors	Page 11 (bottom) and Page 14 (top)
Comparing ratios	Ratio table No ratio tables, but can use arrow notations	Finding common numerators or denominators; size of the scale factors; unit ratios	Pages 18-23
Comparing rates	Ratio table; arrow notation; standard proportion notation	Difference between a ratio and rate; unit rates; common denominator and numerators	Pages 24-27

The launch of the instructional sequence is very important. While the first page is trivial for students to solve, it is critical to spend a few minutes discussing the meaning of students’ symbolizing here. Watch for students to draw lines from one foodbar to a collection of aliens. According to Battista and vanEmeron (XXXX), linking two composite units together is part of the foundation of understanding ratios and rates conceptually. **Iterating composite units** is a conceptual idea that is built during elementary school. For example, understanding 7×5 conceptually means that a student can mentally iterate 5, seven times. If a student can reason that way, they have **iterated** (put together) a **composite unit** of 5, seven times. To iterate **linked composite units**, Battista and

vanEmeron argue that a student has to mentally iterate not just one composite unit, but two composite units that are linked multiplicatively. In other words, understanding that the ratio $2/3$ is equivalent to $10/15$ means that students can iterate a composite unit of 2, five times AND 3 five times. The link is critical and takes on new meaning for students as they wrestle with the different types of instructional activities. For instance, it is fairly easy for students to argue that if 2 food bars can feed 3 aliens, then 10 food bars can feed 15 aliens.

Students may add 2 five times and add 3 five times, add 8 to 3 to get 11 aliens because they added 8 to 2 to get ten, or say 2×5 and 3×5 to get the ratio $10/15$. The student that solves it the first way is explicitly showing that they can iterate two composites (2 and 3) the SAME number of times, five. The 2 and the 3 are linked together so that however many times you add the 2 to itself, you must do the same to the 3.

The student who solves it the second way is not iterating linked composites, they are merely using additive reasoning. The link for these students is additive, what you add to the 2, you must add to the 3.

The third way of reasoning builds of the first way in that the link between the two composites are more explicitly multiplicative; if you have five groups of two, you must also create five groups of three.

The most sophisticated link that students can form is when the two composites are linked multiplicatively. In other words, the equivalence class of ratios has the characteristic that all denominators are 1.5 times as big as the numerator of the ratio. For example, 3 is 1.5 times as big as 2, and so is the case for 10 and 15. Constructing the idea that this is true for all equivalent ratios is fundamental to the concept of ratios and rates. In addition, knowing that this multiplier (1.5) is also the unit rate or the constant of proportionality is important.

The instructional sequence presents problems so that through student solutions and careful guidance, many students can come to understand ratios and rates in this multiplicatively linked way. The teacher's manual has a column that describes the students' solutions we have seen when enacting each page (right side). At the bottom of the page, we have described the big idea or lesson objectives along with the rationale for the problems on the page and some notes to the teacher that can help with guidance during the LED cycles.

LAUNCH



NOTES:

Alien Dream

Begin the unit by telling a story about a scary dream you had last night. The story can resemble the one below or be completely different.

In your dream, a noise awoke you from your sleep and you ventured into the kitchen to investigate the source of that noise. You saw a swarm of aliens eating your food. They saw you and started chanting, “more bread, more bread.” You only had one bread loaf which wasn’t enough to feed the aliens, so they attacked you. You woke up with a new math lesson in mind!

RATIO AND RATES

1. Is there enough food? Explain.



2. Is there enough food? Explain.



3. How many food bars are needed to feed these aliens? Explain.



Anticipated Student Thinking:

NUMBER ONE

- Some students will say not enough food. Draw line from one food bar to 3 aliens but the last 3 do not have a food bar.
- Some will circle the entire collection of 3 aliens and 1 food bar

NUMBER TWO

- There is more than enough food because they draw a line from 3 aliens to 1 food bar until all aliens are used up
- Some will circle the aliens/food bar collection
- Some might say that you could feed 3 more aliens if you had another food bar

NUMBER THREE

- Many students will say that there are 5 food bars needed because they circle groups of 3 aliens,
- They draw a food bar and link it to 3 aliens as many times as they can (5 times)

Big Mathematical Idea(s): The idea of this page is to encourage students to link two composites together.

Rationale: It is an easy page for students so it should only take a couple of minutes as a beginning page. Make sure to highlight the links that students form between a food bar and a composite of aliens, either verbally or with symbols.

Teacher Notes:

LAUNCH: Let students know that that one food bar can feed 3 aliens comfortably. However, if the aliens are not fed adequately, there may be an intergalactic war. You need to ensure that this does not happen. Write some proof on your paper to show that there is enough food or there is more needed.

EXPLORE: About 2-5 minutes

DISCUSSION: The discussion should focus on the link between 3 and 1. Call on students who have drawn some type of symbol that illustrates the link. Ask students explicitly what that line stands for and hopefully they will explain that for every 3 aliens, there is one food bar.

As a follow up question to number 3, you might ask the class what would happen if one more alien were added. Some might say that you would need another food bar, some might say you would only need part of a food bar, some might say $\frac{1}{3}$ food bar. Encourage that 3-1 link.



1. Will 12 food bars be enough to feed 36 aliens? Explain.

2. Will 24 food bars be enough to feed 72 aliens? Explain.

3. Will 6 food bars be enough to feed 18 aliens? Explain.

4. Will 8 food bars be enough to feed 20 aliens? Explain.

5. How many food bars are needed to feed 39 aliens? Explain.

Anticipated Student Thinking:

NUMBER ONE

- 36 divided by 12=3
- Draw 12 food bars with 3 aliens next to each
- Makes a table 1, 3; 2, 6; 3, 9; 4, 12; 5, 15 etc.

NUMBER TWO

- Some might relate this one to number one and double 12, so double 36.
- 72 divided by 24 is 3
- Some might add 12 to 12 to get 24 and add 12 to 36 to get 48
- Some might draw 24 food bars and 72 aliens.
- Make a table again with every value in it
- Continue the picture or table from number one

NUMBER 3

- 18 divided by 6 is 3
- Divide 24 in number 2 by 4, so divide 72 by 4 to get 18
- Draw or make a new table
- Look at their table from #1 or #2

NUMBER 4

- 39 divided by 3 is 13
- Draws 39 aliens and circles 3 at a time
- Makes a table and stops at 13

Big Mathematical Idea(s): The idea of this page is to encourage students to link two composites together and to begin to organize these links when there are large quantities involved.

Rationale: Students need to find a way to organize the links as they increase in size. A ratio table should be introduced from students' work on this page.

LAUNCH: As students to write something down to show how they solved each problem. They need to make sure they feed the aliens appropriately so the intergalactic war does not start.

EXPLORE: 10 minutes

DISCUSSION: Sequence the solutions by having a student who did the division strategy first. The student likely will not be able to explain why they divided. Move to the students who drew the picture, table second, additive reasoning (incorrect) next. Ask students what is common and different about the picture and table strategy. They will likely say that they both take a long time to create but the table is quicker. In fact, name the table method as a ratio table and acknowledge that it is quicker than drawing pictures but that it represents the picture in a more organized way. Do not play up the division strategy today. Let students know that they can use ratio tables or pictures to justify their thinking on future problems since those are the ones that seem to make sense to most students.



1. How many food bars are needed to feed 30 aliens? Explain

2. How many aliens can you feed with 5 food bars? Explain

3. Using a table show how many food bars you would need to feed 70 aliens?

4. How many food bars do we need to feed 35 aliens? Explain.

Anticipated Student Thinking:

NUMBER ONE:

- Students might construct a ratio table and multiply 5 aliens by 6 to get 30 aliens and 1 by 6 to get 6 foodbars
- Some might create a long ratio table 1, 5; 2, 10; 3, 15 etc.
- Some might say 30 divided by 5 is 6

NUMBER TWO:

- Some might look in their long ratio table from number 1 to get 25.
- Others might create an entirely new long table
- Some might create a short table multiplying 1×5 and 5×5
- Some might look at their short table from above and go back 1 food bar and 5 aliens

NUMBER THREE:

- Long table versus a short table for 14 food bars
- Some might use answer for number one 6 food bars for 30 aliens, so 12 food bars for 60 aliens, then add up to 70 aliens

NUMBER FOUR:

- Might start with number one and add one more food bar
- Short or long table

Big Mathematical Idea(s): Begin to reason with short or long ratio table; constructing a build up strategy versus an abbreviated strategy

Rationale: Students should begin to use a table to organize their thinking

Teacher Notes:

LAUNCH: Begin with a warm up...show a long table with the rule 1 food bar feeds 4 aliens, how many aliens to feed 44 aliens? Show a ratio table with 1 and 4 in it and ask students if anyone can go straight to the 44 aliens without having to write all the stops in between. They don't have to do it, but see if anyone can. Have a discussion about what the $\times 11$ means. It is more than the number you multiply by...it is 11 sets of 1 food bar and 11 sets of 4 aliens. Have students use a picture to explain it.

Launch this page and suggest that students use either a short or long ratio table but they should be prepared to explain their thinking.

EXPLORE: about 10 minutes

DISCUSSION: Begin with number one and ask students who drew a picture, drew a long ratio table, and a short ratio table to show their thinking at the board. Ask students to compare the ways. Which one is shorter? What does the $\times n$ mean in the short ratio table? Teacher should facilitate a horizontal scale factor as *groups of* food bars and *groups of* aliens. If anyone constructs an additive strategy, have them present it as a contrast.



1. Will 10 food bars be enough to feed 20 aliens? Explain.
2. Will 12 food bars be enough to feed 22 aliens? Explain.
3. How many aliens can 14 food bars feed? Explain.
4. How many aliens will 98 food bars feed? Explain.
5. How many food bars are needed to feed 16 aliens? Explain.

Anticipated Student Thinking:

NUMBER ONE:

- Some students will circle one food bar and 2 aliens; so 10 food bars will feed 20 aliens
- Some will put it in a ratio table and make a long one to find that it is true
- Others will make a short table
- Some will still be making a drawing

NUMBER TWO:

- Students will put 12 food bars in a ratio table and try to get as close to 22 as possible. They will see they can't.
- Others will find the "unit rate" as 1 for 2 and say that you'd need 11 food bars only

NUMBER THREE:

- Short or long ratio table; either going to unit rate 28 or long table
- Some will say 14 divided by 2 times 14

NUMBER FOUR:

- Short or long table; some unit rates, some not

NUMBER FIVE:

- Short or long tables with unit rate or not

Big Mathematical Idea(s): using non-unit rates to create equivalent ratios; what does a unit rate mean; usefulness of unit rate

Rationale: students might begin to construct unit rate as a mathematical object that can be useful for determining a number of equivalent ratios; students get more facile with short ratio tables

Teacher Notes:

LAUNCH: Bellwork: show a long table that starts at $1/2$; $2/4$; $3/6$; $4/8$; $5/10$; $6/12$. Ask students to predict how many aliens will be fed by 10 food bars; students might continue the long table; some might notice a vertical relationship between the food bars and aliens (there's always twice as many aliens as food bars), so

EXPLORE: about ten minutes

DISCUSSION: pictures, long tables, short tables, tables that go to the unit rate (in that order), have a discussion about unit rate and why it was a good one to go to for these problems. Highlight the unit rate strategy as a useful one and if it feels right, name it "Stephanie's" unit rate or whoever introduced it. Naming it after a person gives ownership and will be remembered more.

How many aliens will 26 food bars feed?



SARAH:

aliens	3	39
FB	2	26

Diagram showing a multiplier of $\times 13$ from 3 to 39 and from 2 to 26.

SAMPSON:

aliens	3	6	9	12	15	18	21	24	27	30	33	36	39
FB	2	4	6	8	10	12	14	16	18	20	22	24	26

SALLY:

aliens	3	27
FB	2	26

Diagram showing an additive multiplier of $+ 24$ from 3 to 27 and from 2 to 26.

STEVE:

aliens	3	39
FB	2	26

Diagram showing a multiplier of $\times 1.5$ from 3 to 39 and from 2 to 26.

WHO DO YOU AGREE WITH AND WHY? It is okay to agree with as many people as you want.

Anticipated Student Thinking:

Almost all students will choose Sarah, Sampson and Steve because each of them gets 36 as the answer. However, they may not know why Sally's does not work and why Steve's DOES work.

Big Mathematical Idea(s): Additive versus multiplicative reasoning/proportional reasoning

Rationale: Students need to analyze the strategies that are correct and incorrect. Also, this is a chance to explore what the scale factors in Steve and Sarah's stands for (aliens always 1.5 times greater than foodbars; unit rate versus 13 groups of aliens). Why can't you add by the same amount?

Teacher Notes:

LAUNCH: Let students know that one of your classes last year solved the problem in four different ways. Which one or ones do they agree with and be prepared to defend the choice.

EXPLORE: 5 minutes

DISCUSSION: Who agrees with Sarah, Sally, Steve, Sampson? What are your rationales? What does Sarah's $\times 13$ stand for? Where does it show up in Sampson's long table? Why can't you just add 24 and 24? Didn't your teacher tell you last year that whatever you do to the top you do to the bottom? What does the $\times 1.5$ stand for in Steve's? Where does it show up in Sampson's long table? Where does it show up in the picture rule?



Two food bars will feed 5 aliens

1. Will 6 Bars feed 15 aliens? Explain.
2. How many aliens will 1 Bar feed? Explain
3. How many aliens will 20 Bars feed? Explain.
4. How many Bars will you need to feed 35 aliens?

Anticipated Student Thinking:

NUMBER ONE:

- Some students will make a short table starting with $2/5$ and multiply both by 3 to see that 6 will feed 15.
- Some students will still draw out a long table

NUMBER TWO:

- Some students will argue that it cannot be done because you cannot feed half an alien
- Some students will say 1 food bar will feed 2.5 aliens.
- Some will be confused

NUMBER THREE:

- Some will make a short table either starting with $2/5$ or $1/2.5$ and scale up to 20 food bars

NUMBER FOUR:

- Some will make a long rate table and go up to 35 aliens
- Some might make a short table and scale up to 35 aliens, either starting with $2/5$ or $1/2.5$ to get 14 food bars.
- Some might accidentally go up to 35 food bars in a short table

Big Mathematical Idea(s): Using non-unit rates to create equivalent ratios; what does a unit rate mean; usefulness of unit rate

Rationale: In this case, the unit rate is a decimal and students will need to think about what that means.

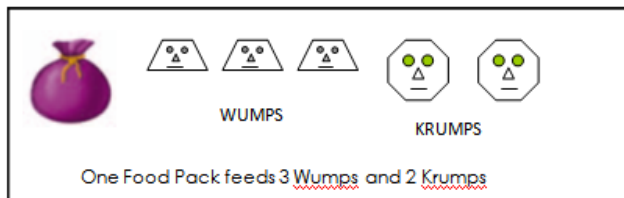
Teacher Notes:

LAUNCH: same as usual

EXPLORE: 10 minutes

DISCUSSION: Ask if anyone had trouble answering question one and make sure they all agree with the answer. Begin discussion with number 2. Some students will argue that you cannot answer this one because it doesn't make sense to feed a half alien. Other students might argue that you can have an alien "half full". Praise this type of thinking and say that we will allow aliens to be partially full from now on.

Finish the page by discussing students' solutions that use the unit rate or not. If someone mixes up 35 food bars with aliens, reiterate that labels in the ratio table are very helpful for organizing work.



1. How many Wumps and Krumps will 5 food packs feed? Explain.
2. How many Wumps and Krumps will 10 food packs feed? Explain.
3. How many Wumps and Krumps will 15 food packs feed? Explain.
4. There are 21 Wumps and 14 Krumps. How many food packs are needed? Explain.
5. There are 24 Wumps and 16 Krumps. How many food packs are needed? Explain.

Anticipated Student Thinking:

NUMBER ONE:

- Students might draw a picture of 5 food packs and 5 sets of 3 wumps/2 krumps to get 15 wumps and 10 krumps
- Some might draw a ratio table with three rows and either do a long or short table

NUMBER TWO:

- Same as above
- Some students may notice that there are twice as many food packs in this question so the wumps and krumps will be doubled as well

NUMBER THREE:

- Same as above
- Some might notice that the food packs keep going up by 5s, so the wumps will go up by another 15 and the krumps will go up by another 10

NUMBER FOUR:

- Short or long tables, pictures
- Some might notice that the wumps are 7 times bigger and the krumps are 7 times bigger, so the food packs must be 7 times bigger

NUMBER FIVE:

- Same as number four

Big Mathematical Idea(s): Linking 3 composite units

Rationale: This page is not necessary, but could be interesting if there is time. This page requires students to coordinate three units simultaneously which could expand their notion of a ratio/rate. Could be used as a challenge page or as homework.

Teacher Notes:

LAUNCH: A different galaxy introduces us to wumps and krumps aliens. In this case one food pack can feed 3 wumps and 2 krumps. Can you figure out the questions below? If you'd like to draw a picture or use a ratio table to show your thinking, please feel free.

EXPLORE: 10-15 minutes

DISCUSSION: Depending on how this paper is used, there may or may not be a discussion. If there is a discussion, focus on the multiplicative relationships and what the scale factors stand for in each problem.

1. Fill in the missing values in each table below.

Food Bars	2	1
aliens	6	?

Food Bars	2	7
aliens	6	?

Food Bars	2	?
aliens	6	7

Food Bars	2	?
aliens	6	1

2. Use a table to find how many food bars are needed to feed 48 aliens.

3. Use a table to find how many food bars will feed 9 aliens.

4. Use a table to find how many food bars will feed 27 aliens.

5. To find out how many food bars will feed 30 aliens, Carol says, "In problem 4, there were 27 aliens and now I have to feed 30 aliens. They just added 3 more aliens so I'll just add three more food bars!"

Is Carol right or wrong? Explain.

Anticipated Student Thinking:

NUMBER ONE:

- Some might multiply $\times 3$ vertically or divide by an appropriate scale factor horizontally
- Some might give up because either the scale factor is a decimal or the resulting food bar or alien is a decimal
- Some might use additive reasoning here

NUMBER TWO:

- Some might start from unit rate and scale up horizontally or vertically
- Some might scale up from $2/6$ horizontally or vertically
- Some might get 48 aliens reversed with 48 food bars

NUMBER THREE:

- Some might start with the unit ratio to get 3 food bars
- Some might start from $2/6$ and multiply either horizontal or vertically to get 3
- Some might start with $2/6$ and add 3 to each to get 5 food bars

NUMBER FOUR:

- Same as above
- Some might take their answer from number 3 and triple it

Big Mathematical Idea(s): explore the meaning of a decimal scale factor; explore additive versus multiplicative reasoning again

Rationale: students should be fairly comfortable with solving alien problems. Introducing decimal scale factors might bring some of them back to additive reasoning

Teacher Notes:

LAUNCH: I decided to give you problems in table form now. See if you can figure out some of these tough ones.

EXPLORE: 10-15 minutes

DISCUSSION: Begin with problem one and discuss vertical and horizontal methods. Many students will suggest that the "rule" is missing but then others will counter that the rule can be found in the table...1 food bar can feed 3 aliens. Really dig in on the tables that have decimal scale factors. What do those mean? Which way is easier, horizontal or vertical? Sometimes one is easier to calculate than the other. Make sure you discuss number 5 to bring out the meaning of additive versus multiplicative again. Be careful...some students may have formed a very procedural understanding of this..."whatever you do to the top you do to the bottom, unless it is addition and subtraction". If needed, go back to a simple ratio situation to explore this in a more conceptual way.

1. Fill in the missing values in each table below.

Food Bars	3	6
aliens	5	?

Food Bars	3	9
aliens	5	?

Food Bars	3	10.5
aliens	5	?

Food Bars	3	?
aliens	5	2.5

2. Use a table to find how many food bars are needed to feed 90 aliens.

3. Use a table to find how many food bars will feed 150 aliens.

4. Use a table to find how many food bars will feed 75 aliens.

5. Use a table to find how many aliens can be feed with 48 food bars.

Anticipated Student Thinking:

NUMBER ONE:

- Horizontal scale factors
- Vertical scale factors
- Additive reasoning

NUMBER TWO:

- Use $\frac{3}{5}$ to scale up horizontally to get 90 aliens 54 food bars
- Use unit rate

NUMBER THREE:

- Most popular will be horizontal scale factors

NUMBER FOUR:

- Same as above

NUMBER FIVE:

- Same as above
- Some might confuse 48 food bars with aliens

Big Mathematical Idea(s): determining the most useful scale factor (horizontal vs. vertical); additive reasoning

Rationale: students need more practice with decimal scale factors and determining which one will be easier (vertical or horizontal); help them become more flexible with choosing

Teacher Notes:

LAUNCH: same as before

EXPLORE: 10-15 minutes

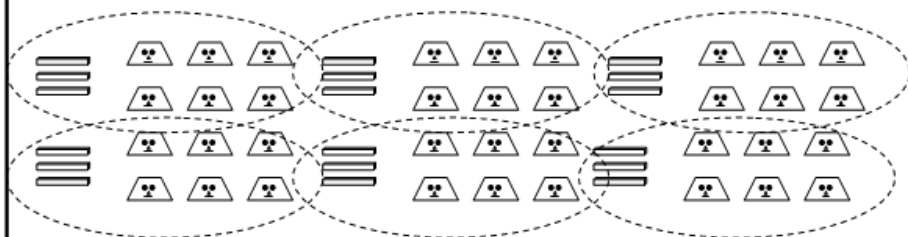
DISCUSSION: highlight solutions that used easy scale factors rather than decimals

Anticipated Student Thinking:



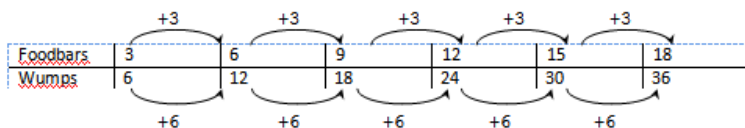
How many Wumps can be fed with 18 food bars?

JANE'S SOLUTION:



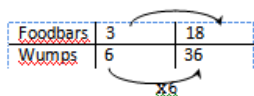
Jane says, "I drew a picture. Every time I added 3 foodbars, I added 6 Wumps. So, $3+3+3+3+3+3=18$ food bars because there are three in each circle. Then, $6+6+6+6+6+6=36$ Wumps since there are 6 Wumps in each of the 6 circles."

JACK'S SOLUTION:



Jack says, "I started with the 3 foodbars and 6 Wumps. Every time I added 3 foodbars, I added 6 Wumps. I stopped when I got to 18 food bars and saw that I had 36 Wumps."

JILL'S SOLUTION:



Jill says, "I started with 3 foodbars and 6 Wumps. I took a shortcut and instead of adding 3 six times and adding 6 six times, I multiplied both times 6 to get my answer."

Sally says that all three of these give the same answer. She also says that Jack's is the same as Jane's but faster, and that Jill's is the same as both of them, just faster than both. Do you agree with Sally? Explain.

Big Mathematical Idea(s): Assessment of how students understand the nature of the scale factors and the procedures they have used with ratio tables

Rationale: what sense do students have of their actions so far

Teacher Notes:


LAUNCH: 2 minutes...quiz or homework

EXPLORE: 5-10 minutes


DISCUSSION: No discussion. This is an assessment. You might come back to discuss this page if students show you that the discussion is needed.


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General Requirements

- ✓ Valid license posted for parents to see.
- ✓ All staff appropriately screened.
- ✓ Maintain minimum staff-to-child ratios:

Under 1 yr. old	1:4
1 yr. old	1:6
2 yrs. old	1:11
3 yrs. old	1:15
4 yrs. old	1:20
5 yrs. old & older	1:25
- ✓ Maintain appropriate transportation vehicles (if transportation is provided).

- ✓ Provide parents with written disciplinary practices used by the facility.
- ✓ Provide access to the facility during normal hours of operation.

Physical Environment

- ✓ Maintain sufficient usable indoor floor space for playing, working, and napping.
- ✓ Provide space that is clean and free of litter and other hazards.
- ✓ Maintain sufficient lighting and inside temperatures.
- ✓ Equip with age and developmentally appropriate toys.
- ✓ Provide appropriate bathroom facilities and other furnishings.
- ✓ Provide isolation area for children who become ill.
- ✓ Practice proper hand washing, toileting, and diapering activities.

Training Requirements

- ✓ 40-hour introductory child care training.
- ✓ 10-hour in-service training annually.
- ✓ 0.5 continuing education unit of approved training or 5 clock hours of training in early literacy and language development.
- ✓ Director Credential for all facility directors.

Health Related Requirements

- ✓ Emergency procedures that include:
 - Posting Florida Abuse Hotline number along with other emergency numbers.
 - Staff trained in first aid and CPR on the premises at all times.
 - Fully stocked first aid kit.
 - A working fire extinguisher and monthly fire drills with children and staff.
- ✓ Locked storage place for storing medication and hazardous materials.

Food and Nutrition

- ✓ Post a meal and snack menu that provides daily nutritional needs of the children (if meals are provided).

Record Keeping

- ✓ Maintain accurate records that include:
 - Children's health exam/immunization record.
 - Medication records.
 - Enrollment information.
 - Personnel records.
 - Daily attendance.
 - Accidents and incidents.
 - Parental permission for field trips and medications.

Tiny Tots

Tiny Tots Daycare has a teacher to infant ratio of 1:5.

Teachers	1	
Infants	5	

- How many teachers must be in the room if there are 25 infants? Explain.
- What is the maximum number of infants that can be in the room if there are 6 teachers? Explain.
- Tiny Tots Daycare has a toddler to teacher ratio of 8 to 1. Use a ratio table to determine how many teachers can be in the room if there are 24 toddlers. Show your work and explain.
- Decide which of the ratios below belongs in the Teacher/Infant table and which belong in the Teacher/Toddler table.

2/16	4/20	80/400	2:10
8/64	6 to 48	7:35	7/56

Anticipated Student Thinking:

NUMBER ONE:

- Not much diversity

NUMBER TWO:

- Either 5 x 6 because of 1 x 6 or 25 divided by 5.

NUMBER THREE:

- Many students will use horizontal or vertical scale factors
- Some students will get confused with the 8 coming first

NUMBER FOUR:

- Many students will "reduce" each ratio to the unit ratio and compare
- Other students will see if the denominator is divisible by 8 or 5. Those that are divisible by both are problematic
- Other students will compare the numerator to the denominator and find a vertical scale factor (e.g., 2×8 is 16)

Big Mathematical Idea(s): reasoning about equivalent ratios in a new context

Rationale: can student reason proportionally in a new context? Compare equivalent ratios. Learn the terminology and conventions of ratios, the way they are worded as well as the way they are written.

Teacher Notes:

LAUNCH: Give each student a day care brochure provided. Ask them to look closely at General Requirements section and discuss what those numbers mean. We have been studying ratios in this class...any time we compare two amounts like food bars to aliens or infants to teachers, that is called a ratio. Add to word wall. Ask students what the ratio of teacher to student in your classroom is and write it on the board. Tell students that you want this next page will ask them questions about a fictional daycare called Tiny Tots.

EXPLORE: 10-15 minutes

DISCUSSION: Not much discussion on numbers 1 and 2. Some discussion might occur on 3 in terms of how to set up a ratio table and the way a ratio is written. Number four is the most interesting one, so have a variety of student thinking presented. No order necessary.

Anticipated Student Thinking:**NUMBER ONE:**

- Horizontal or vertical scale factors
- Add one for and 5 against
- Add one for and 1 against
- Horizontal or vertical scale factors
- Correct/incorrect additive strategy
- Same as above
- Same as above
- Some people might not notice the AGAINST part and miscalculate

NUMBER TWO:

- Same as above for all questions
- From b to c, someone might add one more cup of sugar to get 9 and 2 more flour to get 18

1. Bill conjectured that for every person that is in favor of the Iraq War, 5 people are against it.

For	1	2
Against	5	?

- According to Bill, if 2 people are in favor of the war, how many people are against it?
- If there are 10 people for the war, how many people are against it?
- If there are 7 people for the war, how many people are against it?
- If there are 45 people AGAINST the war, how many are for it?

2. A recipe for chocolate chip cookies calls for 2 cups of sugar and 4 cups of flour.

Sugar		2
Flour		4

- How many cups of flour are needed if we use only one cup of sugar?
- How many cups of flour are needed for a recipe that uses 8 cups of sugar?
- How many cups of flour are needed for a recipe that uses 9 cups of sugar?
- How many cups of flour are needed for a recipe that uses 11 cups of sugar?
- How many cups of SUGAR are needed for a recipe that uses 12 cups of flour?

Big Mathematical Idea(s): reasoning about equivalent ratios in a new context

Rationale: can student reason proportionally in a new context?

Teacher Notes:

LAUNCH: 1 minute

EXPLORE: 10 minutes

DISCUSSION: Have students present the each strategy listed above. Begin with additive strategy that is wrong. Move to the horizontal and vertical strategies last. Ensure that the last one in each section is discussed to make sure students notice the change in what is asked for.

At LCMS, 75 7th graders prefer action movies to 90 8th graders. Write a ratio table to figure out:

- a. How many 8th graders prefer action movies if 25 7th graders prefer them?
- b. How many 8th graders prefer action movies if 10 7th graders prefer them?
- c. How many 7th graders prefer action movies if 120 8th graders prefer them?

At LCMS, 80 7th graders prefer Fergie to 64 8th graders. Write a ratio table to figure out:

- a. How many 7th graders prefer Fergie if 120 8th graders prefer them?
- b. How many 8th graders prefer Fergie if 10 7th graders prefer them?
- c. How many 7th graders prefer Fergie if 16 8th graders prefer them?

Anticipated Student Thinking:

NUMBER ONE:

- Most likely use the horizontal scale factor
- Some try the vertical SF
- Some find the unit ratio of 1/1.2 and scale up

- Those who find the unit ratio scale up fairly easily
- Other try the HSF or VSF but with difficulty because of the decimals
- Others use an additive strategy incorrectly

- Might use answer from b to scale up quickly
- HSF and VSF or use of unit ratio

NUMBER TWO:

- Same as above

Big Mathematical Idea(s): creating equivalent ratios from beginning ratios that involve large numbers

Rationale: how will students deal with ratios that begin really large? Will they scale down appropriately with, sometimes, decimal scale factors or will they find the unit ratio first?

Teacher Notes:

LAUNCH: Survey was taken at LCMS (or whatever your school is named). Here are some of the results.

EXPLORE: 10-15 minutes

DISCUSSION: Discuss the first problem that has disagreement over the answers. Have students share various strategies and have them decide which strategies are both correct and easy.

1. Which ratios belong to the same ratio tables?

$1/2$

$2/3$

$3/4$

$6/9$

$20/40$

$4/9$

$20/45$

$9/12$

2. For every 3 shirts Marci sells, she earns \$4. How much money will she earn if she sells 4 shirts?

Alex and Ann each solve this problem. Alex says the answer is \$5.33 and Ann says it is \$5. Their work is shown below. Who do you agree with and why?

	ANN'S METHOD				ALEX'S METHOD			
Shirts	3	4			3	4		
\$\$	4	5			4	5.33		

In Ann's method, an arrow labeled "+1" points from 3 to 4 in the Shirts row, and another arrow labeled "+1" points from 4 to 5 in the \$\$ row.

In Alex's method, an arrow labeled "X 1 1/3" points from 3 to 4 in the Shirts row, and another arrow labeled "X 1 1/3" points from 4 to 5.33 in the \$\$ row.

Anticipated Student Thinking:

NUMBER ONE:

- Some might put $\frac{1}{2}$ with $\frac{2}{3}$ because the numerator and denominator are 1 apart in each ratio
- Some might reduce all ratios to lowest terms and see which ones match
- Create a unit rate in each one and match
- Find the VSF in each to see if they are the same
- Some might put $\frac{6}{9}$ and $\frac{9}{12}$ in same table because the numerators are 3 apart from each other and so are the denominators
- Some might make ratio tables to compare two at a time and find that either the SFs match or do not

NUMBER TWO:

- Some will pick Ann's and some Alex's

Big Mathematical Idea(s): Determining equivalent ratios; multiplicative versus additive reasoning with decimal scale factors

Rationale:

Teacher Notes:

LAUNCH: 1 minute

EXPLORE: 5-10 minutes

DISCUSSION: For number one, it will be important to discuss students' strategies. You might start the discussion with $\frac{1}{2}$...which ones would show up in that table? Have students debate why $\frac{2}{3}$ would not if anyone put that. If necessary, fall back on aliens and food bars picture to support students' arguments. Go through the others highlighting the various strategies above.

On number two, you might draw out T-shirts and dollars (3 shirts, 4 dollar bills) to support a discussion about what the SF versus additive number mean in that context. In Ann's, students should argue that it means 1 shirt costs \$1 but Alex's means 1 shirt costs \$1.33. Which is correct? A drawing might help with determining that solution.

Calculator Costs

Below is a ratio table that illustrates the price for a certain type of calculator.

Number purchased	1	2	3	4	5	10	15	20
FCAT								\$120
Scientific							\$240	
Graphing						\$800		

1. How much does it cost to buy 53 FCAT calculators? How much to buy 27 scientific calculators? How much to buy 9 graphing calculators?
2. How many FCAT calculators can a school buy if it can spend \$390? What if the school can spend only \$84?
3. How many graphing calculators can a school buy if it can spend \$2,500? What if the school can spend only \$560?

Anticipated Student Thinking:

- Some students will find the unit rate for each calculator type and reason from there on all questions
- Some students might take half of 20 to get the price for 10, half for the price for 5 and then split that amount up by five to get each of 1-5. Same for graphing calculator.
-

Big Mathematical Idea(s): Finding missing values in a “proportion”. Although not introduced yet, students are reasoning proportional every time they find a missing value in a table because reasoning proportionally means to set two ratios equivalent.**

Rationale: Reasoning proportionally in different contexts.

Teacher Notes:

LAUNCH: 1-2 minutes

EXPLORE: 15-20 minutes

DISCUSSION: This page tends to be pretty easy for students so it can be used as an assessment/quiz or homework

** This task was adapted from CMP2.

- At a dining room table, there are 3 serving utensils for every 2 plates. If there are 10 plates, how many serving utensils are there?
- Mitch paid \$4.50 for 5 computer disks. How much did he pay for 12 disks?
- If you can buy 2 packs of cookies for \$3, how many packs of cookies can you buy for \$21?
- There are 3 boys for every 4 girls in Mrs. Smith's class. If there are 28 students in the class, how many girls and how many boys are there?

Anticipated Student Thinking:**NUMBER ONE:**

- HSF or VSF
- Drawings

NUMBER TWO:

- HSF
- VSF easier
- Drawings

NUMBER THREE:

- Same as number one

NUMBER FOUR:

- Some students do not notice that the 28 refers to students not girls or boys
- Some students put $\frac{3}{4}$ in a table and then add up 3s and 4s until they get a column that totals 28
- Some students say that the $\frac{3}{4}$ boy/girl ratio adds to 7 people total and might even draw a third row in the ratio table for total. Then they scale the 7 up by a HSF of $\times 7$ and scale each individual girl, boy up by 7 each

Big Mathematical Idea(s): reason proportionally, introduce the word and symbols of proportionality

Rationale: Practice situations

Teacher Notes:

LAUNCH: 2 minutes, challenge them on number 4 but don't tell them what the challenge is

EXPLORE: 10-15 minutes

DISCUSSION: This is a good page to introduce typical proportional symbols and the definition. Problems 1-3 are very easy for students, so when discussing number 1, tell students that mathematicians like to make things easy, so instead of using a ratio table, (erase the line in between two ratios in the table) they put an equal sign in between [see example below]

For number four highlight the students who total the students. Distinguish between a ratio and fraction (part/part vs. part/whole)

** These tasks were adapted from a released FCAT test.

Utensils	3	15
Plates	2	x

Diagram showing a ratio table with arrows indicating scaling by $\times 5$ from 3 to 15 and from 2 to x.

Eraser vertical lines and some of horizontal line; put equal sign

Utensils	3	15
Plates	2	x

Diagram showing the ratio table with an equals sign between the two ratios and arrows indicating scaling by $\times 5$.

1. In hot chocolate, for every 2 cups of milk, one needs 4 teaspoons of cocoa. If a person has 5 cups of milk to make hot chocolate with, how many teaspoons of cocoa are needed?

2. These photos are the similar, but one is larger than the other. Explain how you would find the height of the larger photo.



3. Sue can walk 15 miles in 5 hours. How far can she walk in 3 hours?

4. Suppose at UCF there are 4 girls for every 6 boys. There are 250 students altogether. How many boys and how many girls are in this class?

Anticipated Student Thinking:

NUMBER ONE

- HSF or VSF
- Might use a ratio table or a more traditional proportion set up

NUMBER TWO

- Some students draw an arrow from 8 cm to 16 cm and write $\times 2$; then draw arrow from 6 cm to ? and $\times 2$ to get 12 cm
- Some students use the ratio table and use HSF typically
- Some students might add 8 to the 6 cm.

NUMBER THREE

- Same as number 1

NUMBER FOUR

- Might make a ratio table with three rows, the last row containing the total of 10 students and scale up each by 25
- Some will scale up by trial and error until they find the total of 250

Big Mathematical Idea(s): solving missing proportions

Rationale:

Teacher Notes:

LAUNCH: Give a bellwork in purely symbolic form (ex. $2/5 = x/15$)

EXPLORE: 10-15 minutes; challenge them with number 4 again

DISCUSSION: have students show their work, especially highlighting the symbolic solutions. Since number two is an area type problem, students might revert to additive reasoning. Make sure this one discussed if this happens. The pictures are drawn proportionally so that when students say, "I multiplied both by 2" you can ask what the $\times 2$ stands for in this context. It means that two of the short sides create the short side of the larger picture and two of the long sides create the long side of the big picture. Students can show that on the board for students.

Save #4 for last and compare strategies.

Making Blood

The Band wanted to make fake blood for their Halloween party. They found a recipe on the internet that uses water and food dye to make the blood.

Three Band students decided to experiment with the recipe to see who could make the darkest red color blood.

Michelle

8 oz of water
32 drops of red food coloring

Julianna

12 oz of water
24 drops of red food coloring

Kyle

6 oz of water
18 drops of red food coloring

Of the 3 students (Michelle, Julianna, and Kyle), who the darkest blood? Explain!

Anticipated Student Thinking:

- Some students will compare two recipes at a time changing Kyle's to $12/36$ and choosing Kyle; they then compare Kyle to Michelle by changing each to a common numerator of 24 to choose Michelle
- Some students will create common numerators for all three at one time (24) and choose largest number of red drops
- Some students will create unit rates for each one ($1/4$, $1/2$, $1/3$) and choose Michelle. Others might choose $1/2$ because it is the biggest number.
- Some students will just draw an arrow from numerator to denominator and say $\times 4$, $\times 2$, and $\times 3$, so Michelle's
- Some students will find the difference between the numerator and denominator (24, 12, and 12) and decide Michelle for the wrong reason
- Some might say that Michelle's is the most because it has the most red drops (32)

Big Mathematical Idea(s): comparing non-equivalent ratios

Rationale: how will students change their ratios in order to determine which amount to choose?

Teacher Notes:

LAUNCH: 1 minute

EXPLORE: 10-15 minutes

DISCUSSION: Have students debate their choices. Do not highlight the difference method since it coincidentally creates the correct solution. Have the $\times 4$ solutions go last and question the students about what that number means. Acceptable explanations either focus on there are four times as many red drops than water so that is the most. Or they can relate it to the already presented unit ratio strategy.



Fright Night



The LCMS Science Club is hosting Fright Night at Chiles Halloween night. Guests will pay \$2 to walk through their Hall of Horrors. In this Hall of Horrors, the lights will be dimmed and guests will take turns feeling the contents of bowls that are filled with various foods that will feel like body parts. One of the displays in the Hall of Horrors is going to be a Bowl of Intestines. Four members of the Science Club looked on the internet to find recipes for the Bowl of Intestines and found the following:

Catherine's

2 pounds cooked noodles
10 teaspoons Karo syrup

Sean's

1 pounds cooked noodles
4 teaspoons Karo syrup

Caitlyn's

6 pounds cooked noodles
18 teaspoons Karo syrup

Kitty's

8 pounds cooked noodles
48 teaspoons Karo syrup

Anticipated Student Thinking:

Same strategies as last question.

Big Mathematical Idea(s): comparing non-equivalent ratios

Rationale: how will students change their ratios in order to determine which amount to choose?

Teacher Notes:

LAUNCH: 1 minute

EXPLORE: 10-15 minutes

DISCUSSION: Have students debate their choices. Do not highlight the difference method since it coincidentally creates the correct solution. Have the x6 solutions go last and question the students about what that number means. Acceptable explanations either focus on there are four times as many red drops than water so that is the most. Or they can relate it to the already presented unit ratio strategy.

Which recipe is the stickiest!?!?



Pumpkin Pie

Below are the pumpkin and flour portions of four recipes for pumpkin pie. Which pumpkin pie recipe will be the most "pumpkiny" flavor?

Recipe A

2 cups flour
8 cups pumpkin mix

Recipe B

3 cups flour
9 cups pumpkin mix

Recipe C

8 cups flour
8 cups pumpkin mix

Recipe D

4 cups flour
2 cups pumpkin mix

Anticipated Student Thinking:

Same as before except some may say there is a tie between A and B if they reason about the difference

Some might say recipe B because it has the most pumpkin cups

Big Mathematical Idea(s): comparing non-equivalent ratios

Rationale: how will students change their ratios in order to determine which amount to choose?

Teacher Notes:

LAUNCH: 1 minute

EXPLORE: 10-15 minutes

DISCUSSION: Have students debate their choices. Highlight the difference method since recipe A and B have the same difference.

Anticipated Student Thinking:

Same as before but some might say a tie between 1 and 4 because they are both 1.6 scale factor

Crazy Cauldron

Crazy Cauldron Lady wants to make the hottest guy in town, named Lars, fall in love with her this Halloween. However, she is an extremely ugly witch and needs the most help she can get. She has four different love potions to choose from, but needs the strongest love potion she can find to make Lars fall in love. Which of the following love potions will give her the best chance at love?



Lady

<u>Love Potion 1</u>	<u>Love Potion 2</u>	<u>Love Potion 3</u>	<u>Love Potion 4</u>
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5 ounces water 8 love drops	3 ounces water 6 love drops	6 ounces water 9 love drops	3 ounces water 5 love drops
--------------------------------	--------------------------------	--------------------------------	--------------------------------

Big Mathematical Idea(s): comparing non-equivalent ratios

Rationale: how will students change their ratios in order to determine which amount to choose?

Teacher Notes:

LAUNCH: 1 minute

EXPLORE: 10-15 minutes

DISCUSSION: Have students debate their choices. Highlight the difference method since recipe 1, 2 and 3 have the same difference. Also, the one that has the most love power is the one with the smallest difference.

The Orangey Obstacle

The Party Committee is planning the Spring Dance for LCMS. Four LCMS students bring recipes for orange punch. Which is the orangey-est?

Buffy's Punch

2 cups orange juice
3 cups Sprite

Willow's Punch

5 cups orange juice
9 cups Sprite



Xander's Punch

1 cup orange juice
2 cups Sprite

Gile's Punch

3 cups orange juice
5 cups Sprite



Fraction Dilemma

Tara says, " $\frac{1}{2}$ of Xander's punch is Orange Juice."
Oz says, " $\frac{1}{3}$ of Xander's punch is Orange Juice."



1. Which student is correct and why? Explain.
2. Suppose 240 students bought tickets to the dance. How many cups of orange juice and sprite are needed for the dance if the committee uses Gile's Punch?
3. If the committee makes juice for 240 students, how many BATCHES of Buffy's Punch will they need to make if each student drinks $\frac{1}{2}$ cup?

Anticipated Student Thinking:

- Some students will find the unit ratio and choose the biggest number when really it is the smaller number this time when the 1 is in the numerator
- Some will find the unit ratio
- Some might pick based upon the differences

Big Mathematical Idea(s):

Rationale:

Teacher Notes:

LAUNCH:

EXPLORE:

DISCUSSION:

Anticipated Student Thinking:

Rates and Ratios

We have learned that a ratio is a comparison between two numbers. What you may not know is that the two units compared in a ratio are always the same. For example, you compared **cups** of orange juice to **cups** of sprite. You compared **number** of 8th graders to **number** of 7th graders (both are numbers of people). Rates are ratios that compare two **different** units. For example, when you compared **number** of calculators with their **price**, price and number are two different units. This is called a rate. When your parent drives 65 miles per hour, her speed is called a rate because it compares **miles** to **hours**.

Suppose Sasha travels from Orlando to Tampa. If you have ever made this trip, you know that the traffic is bad on some parts of I-4 and not so bad in other spots. So, sometimes Sasha has to slow down and sometimes he can go fast. He stopped three times to record his time and distance:

- Stop One: 5 miles in 20 minutes
- Stop Two: 8 miles in 24 minutes
- Stop Three: 15 miles in 40 minutes

You may use a rate table to answer the questions below.

On which part was Sasha traveling fastest? Slowest?

Big Mathematical Idea(s):

Rationale:

Teacher Notes:

LAUNCH:

EXPLORE:

DISCUSSION:

Anticipated Student Thinking:

1. Publix has avocados on sale at 10 for \$2. What is the price per avocado?

- a. How many avocados can you buy for \$10?
- b. How many avocados can you buy for \$75?
- c. How much would 52 avocados cost?

2. Who is the fastest walker?

Montel walks 3 miles in 1 hour.

Jerry walks 6 miles in 2 hours.

Meghan walks 6 miles in 1.5 hours.

Ellen walks 9 miles in 2 hours.

3. Target sells 5 CDs for \$49.95 and Wal-Mart sells CDs at 7 for \$65.80. Which store has the lower price per CD?

4. Which of the situations below are rates and which are ratios?

- a. 3 girls to 4 boys
- b. 75 meters per second
- c. 32 yards to 18 yards
- d. 17 oranges per square feet

Big Mathematical Idea(s):

Rationale:

Teacher Notes:

LAUNCH:

EXPLORE:

DISCUSSION:

Carl has already started Christmas shopping for his family and friends. But he wants to get the best deals possible since he doesn't have much money. He does some online research to compare prices of several gifts he's interested in purchasing. Here are the results:

	TARGET	K-MART	WAL-MART
Item			
underwear	2 for \$8	3 for \$18	10 for \$50
video games	\$25 for 1	\$60 for 3	\$132 for 6
books	4 for \$11.99	11 for \$38.50	7 for \$17.50
X-mas ornaments	12 for \$3.60	4 for \$92	6 for \$2.10

1. Which is the best buy for underwear? Target, Wal-mart or K-Mart? Explain.
2. Which is the best buy for video games? Target, Wal-mart or K-Mart? Explain.
3. Which is the best buy for books? Target, Wal-mart or K-Mart? Explain.
4. Which is the best buy for Christmas Ornaments? Target, Wal-mart or K-Mart? Explain.

Anticipated Student Thinking:

Big Mathematical Idea(s):

Rationale:

Teacher Notes:

LAUNCH:

EXPLORE:

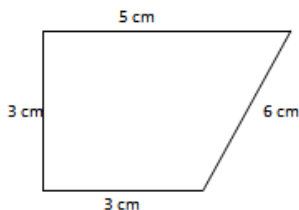
DISCUSSION:

Anticipated Student Thinking:

1. On a map of Florida, 1 inch represents 12 miles in actual distance. What would be the actual distance between Oviedo and Tampa if they are 9 inches apart on the map?

inches	1	
miles	12	

2. On the right is a scale drawing of a pool that is going to be built by Pools-R-Us pool company. Obviously, this is not the actual size of the pool. $\frac{1}{2}$ cm on the drawing stands for 2 feet in real life. Find the dimensions of the actual pool.



3.
4. If 1 gram = 1000 milligrams, how many grams = 2500 milligrams?

5. Sandy mixed 8 ounces, 16 ounces, and 24 ounces of paint together in a bucket. How many cups did Sandy mix in the bucket? 1 cup = 8 ounces

6. If 3 feet = 36 inches, how many inches will 16.5 feet be?

7. A car traveling at a certain speed will travel 90 feet per second. How many yards will the car travel in 90 seconds?

feet		
second		

Big Mathematical Idea(s):

Rationale:

Teacher Notes:

LAUNCH:

EXPLORE:

DISCUSSION:

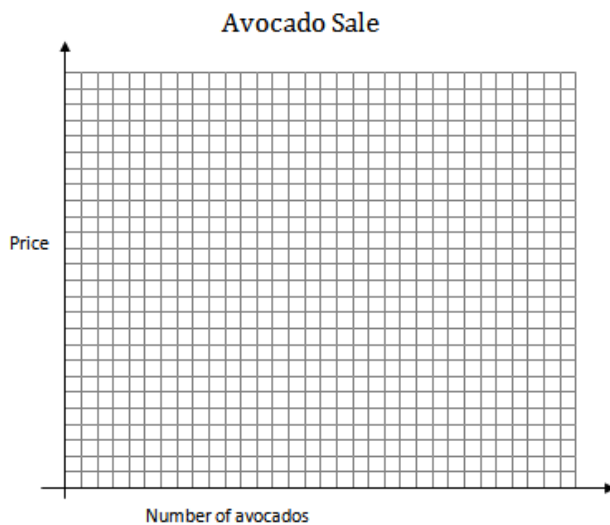
Avocado Sale

A. Publix has avocados on sale at 10 for \$2. What is the price per avocado?

B. Make a table showing the relationship between the number of avocados and the price:

Number avocados	0	1	2	3	4	5	6	7	8
price									

C. Make a graph of this relationship:



Anticipated Student Thinking:

Big Mathematical Idea(s):

Rationale:

Teacher Notes:

LAUNCH:

EXPLORE:

DISCUSSION:

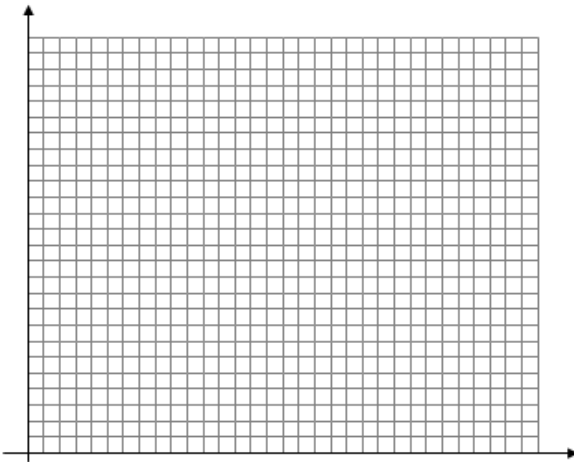
Ahoy Matey!

a. Long John Silver estimates that the 5 treasure chests his pirates stole contain \$370. If the relationship between treasure chests and money is proportional, how much money would be in one chest?

b. Make a table to show this relationship:

	0	1	2	3	4	5	6	7	8

c. Graph the relationship between money and chest;



Anticipated Student Thinking:

Big Mathematical Idea(s):

Rationale:

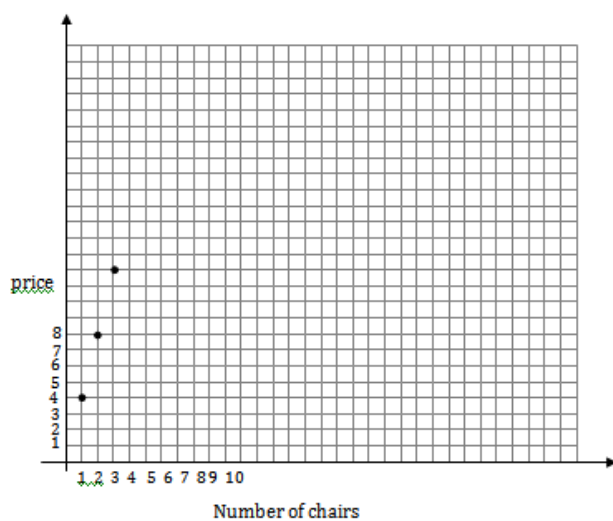
Teacher Notes:

LAUNCH:

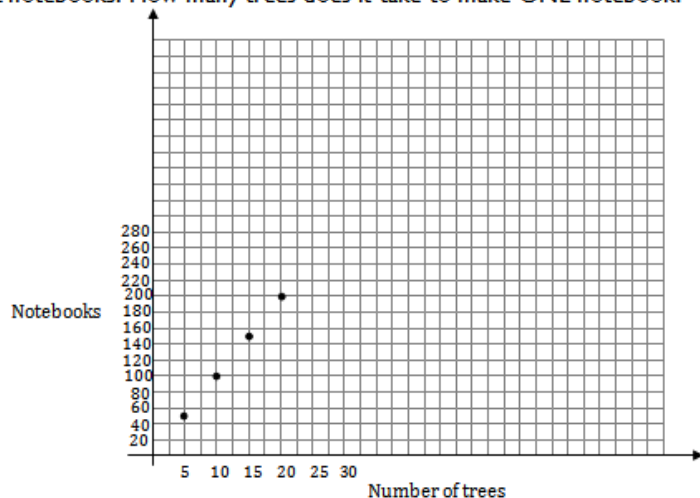
EXPLORE:

DISCUSSION:

The graph below shows the relationship between the number of chairs that a company can build and the price. How much does it cost to make ONE chair?



The graph below shows the relationship between the number of trees it takes to make notebooks. How many trees does it take to make ONE notebook?



Anticipated Student Thinking:

Big Mathematical Idea(s):

Rationale:

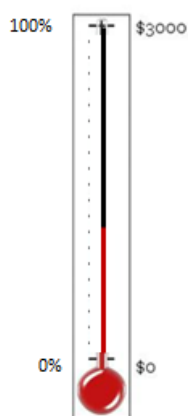
Teacher Notes:

LAUNCH:

EXPLORE:

DISCUSSION:

C C Griffin Fundraiser



- Has C C Griffin reached 50% of their goal? Explain.
- How much money would CC Griffin have raised if they reach 50% of the goal? Explain.

Anticipated Student Thinking:

Students will mark as many halves as they can (50%/\$1500; 25%/\$750; 12.5%/\$375, 6.25%/\$187.25 etc.) (Challenge students to find others besides repeated halving)

Some students might find 10% either by dividing 50% by 5 or by dividing 100% by 10.

Some students might use 50% and 25% to find 75% (\$1500 + \$750)

Others might find 1% by dividing by 100.

Big Mathematical Idea(s): Reason proportionally with percentages and money.

Rationale: This is the introduction to percents as rates. The intent is to build off students' reasoning with rates in the previous section to reason flexibly with percents.

Teacher Notes:

LAUNCH: This task can be launched in a variety of ways. Teachers can bring in real world examples of fundraising drives that utilize some type of fundraising gauge. Red Cross comes to mind or perhaps the school has recently run a fundraiser. Ask students if they have ever seen a gauge like this, where and what they mean. Show the picture of the C C Griffin gauge. Ask the class if C C Griffin has reached 50% of their goal and then how much money they would have earned. Almost all students will answer no; have students come up to explain their reasoning (expect some students to estimate $\frac{1}{2}$ on the bar above and show that it is not quite half. Ask students how they figured out \$1500. Many will say they cut it in half. What did students cut in half? How did they know to cut it in half? When students say that they knew to cut it in half because they needed half of 100% to get to 50%, the teacher should draw an arrow from 100% down to 50% with a $\div 2$ symbol on the arrow and the same $\div 2$ arrow from \$3000 to \$1500 on the right side. Some students might remark that this reminds them of the ratio table. Capitalize on this contribution.

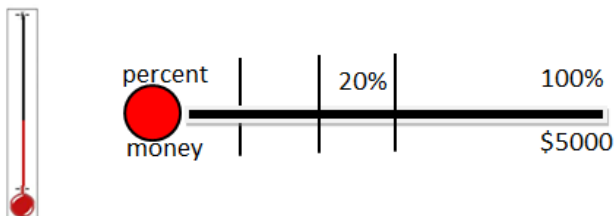
EXPLORE: Tell students that they have already marked 50% on the class gauge. Challenge them to mark four more percents and find the money for them on their own gauge.

DISCUSSION: Begin the discussion by having students show a variety of markings. Begin with a couple of correct ones, making sure to emphasize how the student knew what to divide by using arrows. You might also ask students what are some easy ones to find. They might suggest 10% is easy to find, 1% or 20%). The big idea that should arise in conversation here is the proportional moves from percents and money amounts (divide by 2 on both sides, e.g.). Also highlight flexible strategies if students use them (e.g., if I know 10% is \$300, then I can find 90% by subtracting \$300 from \$3000).

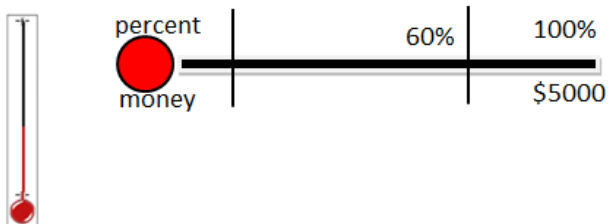
Percents as Ratios

The PTSA's fundraising goal this year is \$5,000. In October they had reached 20% of their goal. How much money have they made so far?

Percent Table



In March they had reached 60% of their goal. How much money have they made so far?



Anticipated Student Thinking:

Students may wish to use the gauge

Question 1:

Students may go from 100 to 1% (\$50) and then scale up to 20% (\$1000)

Others may scale right down to 20% by dividing by 5s.

Others may go to 10% and then to 20%

Others may argue that 5000 is 50 times bigger than 100% so multiply 20% by 50 to get \$1000 (vertical scale factor).

Question 2:

Students might go straight to 60% with a scale factor of 1.666666666...

Go to 1% or 10% and up by SF 60 or 6

Vertical SF of 50

Big Mathematical Idea(s): Reason proportionally with percentages and money.

Rationale:

Teacher Notes:

LAUNCH: If a student remarked that the fundraising gauge was just like the ratio table, compliment them and let them know that the people who wrote this page had the same thought only they took the fundraising gauge and rotated it sideways to look like a ratio table. Use the table/gauge to answer each of the questions about the PTSA's (Parent/Teacher/Student Association) fundraising activities.

EXPLORE:

DISCUSSION: Discussion should be a sharing of the flexible ways students find the percentages. Keep in mind that the ways that are selected should be relatively efficient methods.

We Will Rock You!



The Band wants to go to the State Competition in May but needs to raise \$12,000. How much money will they have if they raise 15% of their goal?



What percent will the Band have collected if they have \$9000?



What percent will the Band have collected if they have \$14,400?

**Anticipated Student Thinking:**

Students may wish to use the gauge

Questions 1 & 2:

Various strategies that resemble the activity sheets previously.

Answers: \$1800 and 75%

Question 3:

Students may say that this cannot be done because there is more money than 12,500.

Other students may figure the amount out by scaling a variety of ways.

Big Mathematical Idea(s): Reason proportionally with percentages and money.

Rationale: This is the first page that has a percent increase. Students might have questions about number three, but do not discuss it with them until the whole class discussion. Let the students analyze their answers and give reasons why the percentage is higher than 100 and what that means.

Teacher Notes:

LAUNCH: Use the table/gauge to answer each of the questions about the Band's fundraising activities. You might find the third question a bit challenging and I can't wait to see what sense you make out of that one.

EXPLORE:

DISCUSSION: Discussions about 1 and 2 should be a sharing of the flexible ways students find the percentages. Keep in mind that the ways that are selected should be relatively efficient methods.

Question 3 is the most critical on the page. It is the first introduction of percentages over 100. Ask students to share the answers they got and then focus the discussion on what that means. What does it mean that they raised 120%? Some students will argue that this means they raised 20% more than they expected. You can capitalize on this and call that the 20% an excess amount and that you will be studying these excesses in the next few pages.

The choir's fund-raising goal is \$400.



1. How much money have they collected if they have reached 50%?

2. How much money have they collected if they have reached 25%?

3. How much money have they collected if they have reached 150%?

4. How much money have they collected if they have reached 10%?

5. How much money have they collected if they have reached 110%?

Anticipated Student Thinking:

Various strategies that resemble the activity sheets previously.

Answers:

Question 1: \$200

Question 2: \$100

Question 3: \$600

Question 4: \$40

Question 5: \$440

These problems are sequenced so that students do not need to use the ratio table but can use answers to some questions to find the answers to others. Some students will use a separate ratio table on all (maybe not the first) and some will recognize that Q2 is half Q1; Q3 is 3x Q1; Q5 is \$400 plus Q4.

Big Mathematical Idea(s): Reason proportionally with percentages and money.

Rationale: This page is presented without ratio tables on it to give students the flexibility to use the table symbolization or not.

Teacher Notes:

LAUNCH: Launch is very typical. Let students know that they can use a ratio table or gauge to solve these if they wish but to make sure they record their reasoning on their paper.

EXPLORE:

DISCUSSION: This can probably be a fast page for students to just read off answers and check for understanding. This page can be used as homework or as a formative assessment. If discussed in whole class, make sure you have students who related questions to each other to present as efficient strategies. You might draw students' attention to Q 3 and 5 again.



math club's fund-raising goal is \$8500.



1. How much money have they collected if they have reached 80%?
2. If they have collected \$1020, what percent of their goal have reached?
3. If they have collected \$11,475, what percent of their goal have reached?
4. If they have collected \$17,000, what percent of their goal have reached?
5. How much money have the collected if they have reached 160% of their goal?

Anticipated Student Thinking:

Various strategies that resemble the activity sheets previously.

Answers:

Question 1: \$6800

Question 2: 12%

Question 3: 130%

Question 4: 200%

Question 5: \$9350

Big Mathematical Idea(s): Reason proportionally with percentages and money.

Rationale:

Teacher Notes:

LAUNCH: Launch is very typical. Let students know that they can use a ratio table or gauge to solve these if they wish but to make sure they record their reasoning on their paper.

EXPLORE:

DISCUSSION: Discussion should focus on the strategies that students use to solve the tasks.

Progress or Not?

Each year the Volleyball Team tries to raise \$10,000. Coach Carnes analyzed the Volleyball Team's fundraising record from last year to this year. Last year the Team met their fundraising goal of \$10,000. This year the team raised \$8000.

Percent	100%	
Money	\$10,000	\$8000

Clearly, the Volleyball Team did not reach goal this year. What was their **percent decrease** from last year?

The Chess Club did not their goal this year and had a 5 **percent decrease** (that is, they only raised 95% of what they wanted to make). What was their original fundraising goal from last year if they made only \$114 this year?

Percent		
Money		

The fifth grade raised 140% of their goal. Their original goal was \$2000. What **percent increase** occurred from last year's goal? How much EXCESS money did they raise?

Percent		
Money		

Anticipated Student Thinking:

Question 1:

Some might say that the percent decrease is \$2000.

Others may say 80% and others may say 20%.

Question 2:

Some might say that the original goal was \$109/\$119 by subtracting/adding 5 to \$114.

Others may say 5%.

Others may have difficulty setting up the ratio table with 95% associated with \$114 but those who do will get \$120 as the original amount.

Question 3:

Note that there are two answers to this question. Students might confuse the excess money with the percent increase. The answers should be 40%. \$800 excess.

Big Mathematical Idea(s): Reason proportionally with percent increases and decreases.

Rationale:

Teacher Notes:

LAUNCH: Launch is very typical. Let students know that they can use a ratio table or gauge to solve these if they wish but to make sure they record their reasoning on their paper.

EXPLORE:

DISCUSSION: Discussion should focus on the strategies that students use to solve the tasks as well as how students set up the ratio table to make sense of the situation. The teacher can support students' reasoning by encouraging students to label the tops of the columns as original amount and new amount to distinguish between what was made before and what is new.

Percent Increase and Decrease

Eric and Marie decide to sell bottled water to make some money. They originally were going to sell each bottle for \$1.25. But they learned that the local vending machine charges less than that, so they lower their price to \$1.00. What was their percent decrease?

	Original	New
Percent		
Money		

Deanna and Rick want to sell homemade brownies to make some money. They originally were going to sell each brownie for \$.50 but their brownies were so popular that they raised the price to \$.70 each. What was their percent increase?

	Original	New
Percent		
Money		

Anticipated Student Thinking:

Question 1:

Decimals might pose a challenge but the percent decrease is 20%. Students will use a variety of strategies as discussed on previous pages.

Question 2:

Same as above with a 40% increase.

Big Mathematical Idea(s): Reason proportionally with percent increases and decreases.

Rationale:

Teacher Notes:

LAUNCH: Launch is very typical. Let students know that they can use a ratio table or gauge to solve these if they wish but to make sure they record their reasoning on their paper.

EXPLORE:

DISCUSSION: This page can be used as an assessment or in class. In an advanced class, it might be interesting to ask students why a quarter difference in the first problem was a 20% decrease and 20 cents in the second problem was 40% (twice as big, but less money).

Sesame Salaries

Sesame Street Incorporated announced the salaries for each of their employees for next year. Listed below is the salary for each employee in 2013 and their new salary in 2014.

Which character had the biggest percent increase?
Which character had the biggest percent decrease?

Employee	2013 Salary	2014 Salary	Percent increase or decrease
Abby	\$10,000	\$14,000	
Bert	\$40,500	\$45,000	
Cookie	\$50,000	\$45,000	
Oscar	\$60,000	\$80,000	
Ernie	\$75,000	\$67,500	
Snuffle	\$25,000	\$20,000	
Mahna Mahna	\$100,000	\$105,000	

Anticipated Student Thinking:

Students may notice the absolute increase in money and argue that since Oscar had a \$20,000 increase, he should be the highest increase, but it is really Abby with a 40% increase even though she only added \$4000.

Same could be argued for Ernie and Mahna Mahna.

Some students will find the percent increase and decrease and argue that Snuffle had the biggest percent decrease with Abby the biggest percent increase.

Big Mathematical Idea(s): Reason proportionally with percent increases and decreases.

Rationale:

Teacher Notes:

LAUNCH: Launch is very typical. Let students know that they can use a ratio table or gauge to solve these if they wish but to make sure they record their reasoning on their paper.

EXPLORE:

DISCUSSION: This discussion can be started by getting students' answers on the board. Students might argue that the biggest increase is Oscar versus Abby. Students should present the arguments for their cases, focusing on the percent versus salary increase. The same can be said for the percent decrease.

1. Mary bought a shirt on Friday that cost \$35. The shirt went on sale two days later for 30% off of the original price. What is the sale price of the shirt?

Percent		
Money		

2. Juan bought a pair of pants that were on sale for 20% off of the original price. He paid \$39.20 for the pants. How much was the original price?

Percent		
Money		

3. Elise and Sue went to dinner. They gave the waiter a 15% tip. The amount of the tip was \$4.50. How much was the bill?

Percent		
Money		

4. Elton paid 116.10 for a pair of in-line skates that were on sale. The original price of the skates was \$135. What percent of the original price is the sale price?

Percent		
Money		

Anticipated Student Thinking:

The biggest difficulty students will have on these is deciding which percent goes with which money amount.

Big Mathematical Idea(s): Reason proportionally with sales price and discounts.

Rationale:

Teacher Notes:

LAUNCH: Launch should include a realistic shopping scenario in order to get vocabulary words of discount percent, discount price, original price, original percent (always 100%), and sales percent, sales price. Let students know that they can use a ratio table or gauge to solve these if they wish but to make sure they record their reasoning on their paper. You might even suggest that they label their columns on ratio tables to keep track of which amount is which.

EXPLORE:

DISCUSSION: Have students present a variety of strategies, in particular those involving stating what each amount stands for in the table. The teacher should also support students by requesting these headings during explanations.

a. What is 43% of 150?

Percent		
Number		

b. What is 86% of 150?

Percent		
Number		

c. 50 is what % of 500?

Percent		
Number		

d. 50 is what % of 1000?

Percent		
Number		

e. 25 is what % of 500?

Percent		
Number		

Anticipated Student Thinking:

The biggest difficulty students will have on these is deciding which percent goes with which money amount.

Big Mathematical Idea(s): Reason proportionally with sales price and discounts.

Rationale:

Teacher Notes:

LAUNCH: Launch should include a discussion of what number would stand for 100% in these kinds of problems. Students should know that they can use a ratio table or gauge to solve these if they wish but to make sure they record their reasoning on their paper. **EXPLORE:**

DISCUSSION: Have students present a variety of strategies, in particular those involving stating what each amount stands for in the table. The teacher should also support students by requesting these headings during explanations.